$$\left(\frac{\partial f}{\partial x}\right)^2 = \left(f_x\right)^2$$

$$\frac{\partial^2 f}{\partial x^2} \rightarrow \frac{\partial^2 f}{\partial x \partial x} = f_{xx}$$

$$\| \nabla f \| = \| \langle f_x, f_y \rangle \| = f_x^{\,2} + f_y^{\,2}$$

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 f}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{1}{r}\frac{\partial f}{\partial r}$$

$$f(x, y) = f\left(x(r,\theta), y(r,\theta)\right)$$

$$\left.\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}\right\} \quad \begin{aligned} \frac{\partial x}{\partial r} &= \cos\theta \qquad \frac{\partial y}{\partial r} = \sin\theta \\ \frac{\partial x}{\partial \theta} &= -r\sin\theta \qquad \frac{\partial y}{\partial \theta} = r\cos\theta \end{aligned}$$

$$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial r}$$

$$\frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial \theta}$$

$$\left[\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta\right]$$

$$\frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial x}(-r\sin\theta) + \frac{\partial f}{\partial y} r\cos\theta$$

$$\frac{\partial^2 f}{\partial r^2} = \frac{\partial}{\partial r}\frac{\partial f}{\partial r} = \cos\theta\,\frac{\partial}{\partial r}\frac{\partial f}{\partial x} + \sin\theta\,\frac{\partial}{\partial r}\frac{\partial f}{\partial y}$$

$$\frac{\partial}{\partial r}\frac{\partial f}{\partial x} = \frac{\partial^2 f}{\partial x \partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial f}{\partial x\partial y}\frac{\partial y}{\partial r}$$

$$= f_{xx}\cos\theta + f_{xy}\sin\theta$$

$$\frac{\partial}{\partial r}\frac{\partial f}{\partial y} = \frac{\partial f}{\partial x\partial y}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y\partial y}\frac{\partial y}{\partial r}$$

$$= f_{xy}\cos\theta + f_{yy}\sin\theta$$

$$\frac{\partial^2 f}{\partial r^2} = f_{xx}\cos^2\theta + f_{xy}\cos\theta\sin\theta$$

$$+ f_{xy}\sin\theta\cos\theta + f_{yy}\sin^2\theta$$

$$\frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \theta} \qquad \frac{\partial x}{\partial \theta} = -r\sin\theta \qquad \frac{\partial y}{\partial \theta} = r\cos\theta$$

$$= f_x(-r\sin\theta) + f_y\, r\cos\theta$$

$$\frac{\partial^2 f}{\partial \theta^2} = \frac{\partial}{\partial \theta}\left(f_x(-r\sin\theta) + f_y\, r\cos\theta\right)$$

$$\boxed{\begin{aligned}
\frac{\partial f}{\partial \theta \partial x} &= \frac{\partial f}{\partial x \partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y \partial x}\cdot\frac{\partial y}{\partial \theta} \\
&= f_{xx} - r\sin\theta + f_{xy}\, r\cos\theta \\
\frac{\partial f}{\partial \theta \partial y} &= \frac{\partial f}{\partial x \partial y}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y \partial y}\cdot\frac{\partial y}{\partial \theta} \\
&= f_{xy}(-r\sin\theta) + f_{yy}\, r\cos\theta
\end{aligned}}$$

$$\frac{\partial^2 f}{\partial \theta^2} = \frac{\partial}{\partial \theta} f_x(-r\sin\theta) + f_x(-r\cos\theta) + \frac{\partial}{\partial \theta} f_y(r\cos\theta)$$

$$+ f_y(-r\sin\theta)$$

$$= \left(f_{xx}(-r\sin\theta) + f_{xy}\, r\cos\theta\right)(-r\sin\theta)$$

$$+ f_x(-r\cos\theta) + \left(f_{xy}(-r\sin\theta) + f_{yy}\, r\cos\theta\right)(r\cos\theta)$$

$$+ f_y(-r\sin\theta)$$

$$= f_{xx}\cancel{r^2}\sin^2\theta - \cancel{f_{xy} r^2 \sin\theta\cos\theta} - \frac{\cancel{r} f_x \cos\theta}{r} \quad 1$$

$$- \cancel{f_{xy} r^2 \sin\theta\cos\theta} + f_{yy}\cancel{r^2}\cos^2\theta - \frac{\cancel{r} f_y \sin\theta}{r}$$

$$\frac{\partial^2 f}{\partial r^2} = f_{xx}\cos^2\theta + \cancel{f_{xy}\cos\theta\sin\theta} \quad 2$$

$$+ \cancel{f_{xy}\sin\theta\cos\theta} + f_{yy}\sin^2\theta$$

$$\frac{1}{r^2}\frac{\partial^2 f}{\partial\theta^2} + \frac{\partial^2 f}{\partial r^2} = \frac{1}{r^2} + 2$$

$$= f_{xx}\sin^2\theta + f_{xx}\cos^2\theta + f_{yy}\cos^2\theta + f_{yy}\sin^2\theta$$

$$\underbrace{- \frac{f_x\cos\theta}{r} - \frac{f_y\sin\theta}{r}}_{-\frac{1}{r}\frac{\partial f}{\partial r}}$$

$$= f_{xx} + f_{yy} - \frac{1}{r}\frac{\partial f}{\partial r}$$

$$\frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial r^2} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} - \frac{1}{r}\frac{\partial f}{\partial r}$$

$f(x,y)$ but want to look at it in polar coordinates

$\left.\begin{matrix} f_x \\ f_y \end{matrix}\right\}$ are also just functions of x & y, so also can be looked at in polar coordinates

$$\frac{\partial}{\partial \theta} f_x = \frac{\partial f_x}{\partial x} \cdot \frac{\partial x}{\partial \theta} + \frac{\partial f_x}{\partial y} \cdot \frac{\partial y}{\partial \theta}$$

$$= f_{xx} \cdot \frac{\partial x}{\partial \theta} + f_{xy} \cdot \frac{\partial y}{\partial \theta}$$

$$w = f(x, y, z) \qquad y = g(s,t), \qquad z = h(t)$$

$$\frac{\partial w}{\partial s} = \underset{0}{\cancel{\frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial s}}} + \frac{\partial w}{\partial y} \cdot \frac{\partial g}{\partial s} + \underset{0}{\cancel{\frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial s}}}$$

$$= \frac{\partial w}{\partial y} \cdot \frac{\partial g}{\partial s}$$

$$\frac{\partial w}{\partial t} = \underset{0}{\cancel{\frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial t}}} + \underbrace{\frac{\partial w}{\partial y} \cdot \frac{\partial g}{\partial t} + \frac{\partial w}{\partial z} \cdot \frac{\partial h}{\partial t}}$$

$r(s)$ is arc length parametrization for a curve

$$K(s) = \left\| \frac{dT}{ds} \right\| = \boxed{\cancel{\frac{\| r'(s) \times r''(s) \|}{\| r'(s) \|^3}} \quad \cancel{= \frac{1}{v(s)} \left\| \frac{dT}{ds} \right\|}}$$

$$T(s) = \frac{r'(s)}{\| r'(s) \|}$$

$$v(s) = \| r'(s) \|$$

$$K(t) = \frac{\| r'(t) \times r''(t) \|}{\| r'(t) \|^3} = \frac{1}{v(t)} \left\| \frac{dT}{dt} \right\|$$

Hill 18W 10:

$$f(x,y) = \begin{cases} \dfrac{x^2y^2}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

For $(x,y) \neq (0,0)$, $f(x,y)$ is a quotient of polynomials, so it's cont. as long as the denominator is non-zero

$$y = mx \qquad \frac{x^2(mx)^2}{x^2+(mx)^2} = \frac{m^2x^4}{x^2(1+m^2)} = \frac{m^2x^2}{1+m^2}$$

Polar substitution

$$\frac{\cancel{r^2}\cos^2\theta \cdot r^2\sin^2\theta}{\cancel{r^2}} = r^2\cos^2\theta\sin^2\theta$$

$$0 \leq |r^2\cos^2\theta\sin^2\theta| = r^2|\cos^2\theta\sin^2\theta| \leq r^2 \cdot 1$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+y^2} = \lim_{r\to 0} r^2\cos^2\theta\sin^2\theta = 0$$

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$$f(x,y) = \sin(x+y) - \cos(x)$$ Find crit. pts

$$f_x = \cos(x+y) + \sin(x) = 0 =$$

$$f_y = \cos(x+y) = 0$$

$$x+y = \frac{\pi}{2} + k\pi, \quad k \text{ integer}$$

$$\cos(x+y) + \sin(x) = \sin(x) = 0$$

$$x = l\pi, \quad l \text{ integer}$$

$$y = \frac{\pi}{2} + k\pi - x = \frac{\pi}{2} + k\pi - l\pi = \frac{\pi}{2} + (k-l)\pi$$

$$f_{xx} = -\sin(x+y) + \cos(x)$$
$$f_{xy} = -\sin(x+y)$$
$$f_{yy} = -\sin(x+y)$$

$$D = \cancel{\sin^2(x+y)} - \sin(x+y)\cos(x) - \cancel{\sin^2(x+y)}$$
$$= -\sin(x+y)\cos(x)$$

$y = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$ | $y = \frac{3\pi}{2}, \frac{7\pi}{2}, \ldots$

$x = 0, 2\pi, 4\pi, \ldots$: $\cos(x) = 1$, $\sin(x+y) = 1$, $D = -1$ saddle | $\cos(x) = 1$, $\sin(x+y) = -1$, $D = 1$ min

$x = \pi, 3\pi, 5\pi$: $\cos(x) = -1$, $\sin(x+y) = -1$, $D = -1$ saddle | $\cos(x) = -1$, $\sin(x+y) = 1$, $D = 1$ max